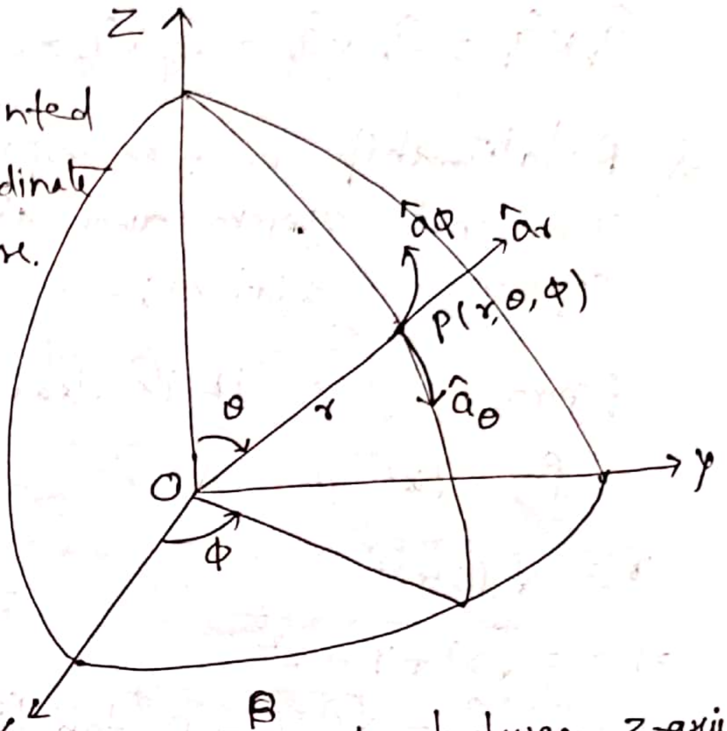
Short notes on spherical coordinate or spherical polar coordinate (r, θ, ϕ).

Spherical coordinate system is most appropriate whenever we dealing with problems having a degree of spherical symmetry.



A point P can be represented as (r, θ, ϕ) in spherical coordinate system as shown in figure. from figure, it is clear that r is the distance of the point P from the origin or the radius of ~~the~~ a sphere centred at origin and passing through the point P; θ (called the colatitude) is the angle between z-axis and the position vector $\vec{OP}$; ϕ (azimuthal angle) is the angle measured from the x-axis in xy plane.

Ranges of the variables r, θ and ϕ in spherical coordinate system are

$0 \leq r < \infty$, $0 \leq \theta \leq \pi$ and $0 \leq \phi < 2\pi$.

A vector $\vec{A}$ in spherical coordinate system may be written as

(A_r, A_θ, A_ϕ) or $\vec{A} = A_r \cdot \hat{a}_r + A_\theta \cdot \hat{a}_\theta + A_\phi \cdot \hat{a}_\phi$ —— (1)

Where $\hat{a}_r$, $\hat{a}_\theta$ and $\hat{a}_\phi$ are unit vectors along r-, θ- and ϕ- directions respectively.

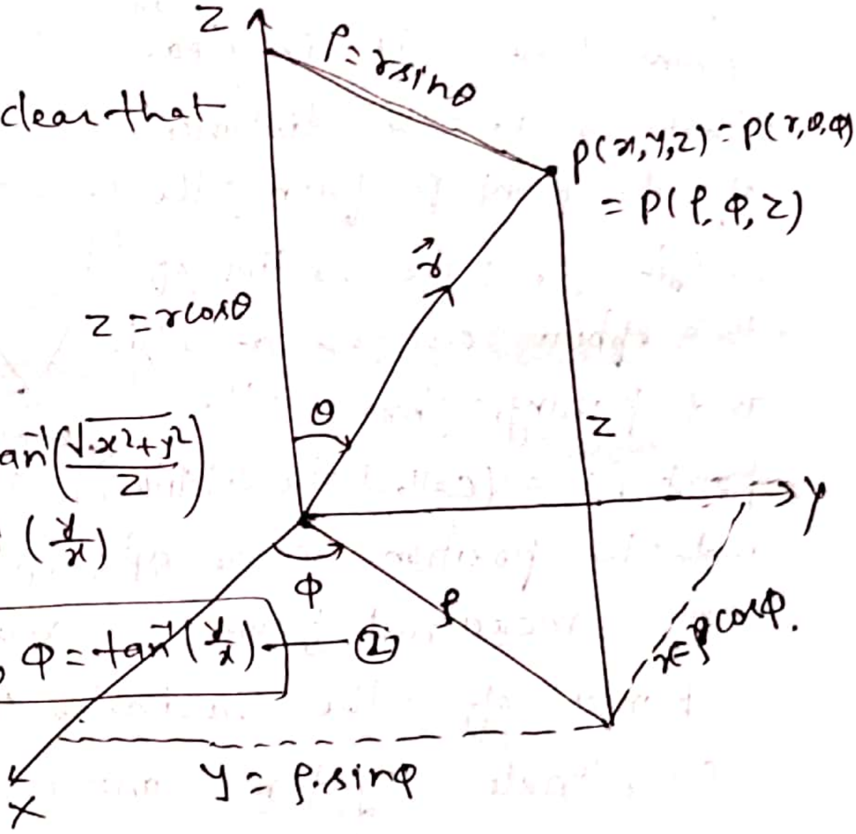
Magnitude of vector $\vec{A}$ is $A = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$

The unit vectors $\hat{a}_r$, $\hat{a}_\theta$ and $\hat{a}_\phi$ are mutually perpendicular unit vectors (orthogonal). $\hat{a}_r$ is directed along radius r or in the direction of increasing r; $\hat{a}_\theta$ is directed along the direction of increasing θ; and $\hat{a}_\phi$ is directed in the direction of increasing ϕ.

$$\hat{a}_r\cdot\hat{a}_r = \hat{a}_\theta\cdot\hat{a}_\theta = \hat{a}_\phi\cdot\hat{a}_\phi = 1, \quad \hat{a}_r\cdot\hat{a}_\theta = \hat{a}_\theta\cdot\hat{a}_\phi = \hat{a}_\phi\cdot\hat{a}_r = 0$$

$$\hat{a}_r\times\hat{a}_r = \hat{a}_\theta\times\hat{a}_\theta = \hat{a}_\phi\times\hat{a}_\phi = 0, \quad \hat{a}_r\times\hat{a}_\theta = \hat{a}_\phi, \ \hat{a}_\theta\times\hat{a}_\phi = \hat{a}_r, \ \hat{a}_\phi\times\hat{a}_r = \hat{a}_\theta$$

* Relationship between space variables (x, y, z) of cartesian coordinate system and space variable (r, θ, ϕ) of spherical coordinate system:



From figure, it is clear that

$$\rho = \sqrt{x^2+y^2}$$

$$r = \sqrt{\rho^2+z^2}$$

$$\Rightarrow r = \sqrt{x^2+y^2+z^2}$$

$$\tan\theta = \frac{\rho}{z} = \frac{\sqrt{x^2+y^2}}{z} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$$

$$\tan\phi = \frac{y}{x} \Rightarrow \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

Thus

$$\boxed{r = \sqrt{x^2+y^2+z^2}, \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right), \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)} \quad (2)$$

Again

$$x = \rho.\cos\phi = r\sin\theta.\cos\phi \quad \because \rho = r\sin\theta.$$

$$y = \rho.\sin\phi = r\sin\theta \sin\phi \quad \because \rho = r\sin\theta$$

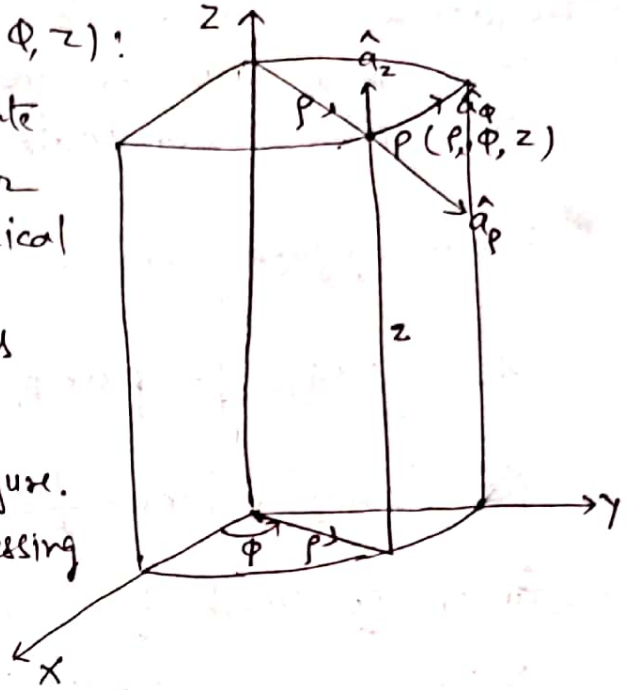
$$z = r\cos\theta$$

Thus

$$\boxed{x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta} \quad (3)$$

Therefore equⁿ (2) is used for transforming the point P from cartesian coordinate (x, y, z) to spherical polar coordinate (r, θ, ϕ) and equⁿ (3) is used for transforming the point P from spherical coordinate (r, θ, ϕ) to cartesian coordinate (x, y, z).

Short notes on cylindrical coordinates or circular cylindrical coordinates (ρ, ϕ, z):



The circular cylindrical coordinate system is very convenient whenever we dealing problems having cylindrical symmetry.

A point P can be represented as (ρ, ϕ, z) in circular cylindrical coordinate system as shown in figure. ρ is radius of the cylinder passing through the point P or radial distance from z axis; ϕ, called azimuthal angle is measured from x-axis in xy plane and z is the same as in the cartesian coordinate system.

The ranges of the variables ρ, ϕ and z in circular cylindrical system are

$0 \le \rho < \infty$, $\quad 0 \le \phi < 2\pi$ $\quad$ and $\quad -\infty < z < \infty$.

A vector $\vec{A}$ in cylindrical coordinate system can be written as (A_ρ, A_ϕ, A_z) or $\vec{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \cdot \hat{a}_z$

where $\hat{a}_\rho$, $\hat{a}_\phi$ and $\hat{a}_z$ are unit vectors in cylindrical coordinate system in ρ-, ϕ- and z-directions respectively as shown in figure. These three unit vectors are mutually perpendicular to one another i.e, they are orthogonal.

The magnitude of $\vec{A}$ is $A = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$.

$\hat{a}_\rho$ is directed in the direction of increasing ρ, $\hat{a}_\phi$ is directed along increasing ϕ and $\hat{a}_z$ along positive z direction.

$$\hat{a}_\rho \cdot \hat{a}_\rho = \hat{a}_\phi \cdot \hat{a}_\phi = \hat{a}_z \cdot \hat{a}_z = 1, \quad \hat{a}_\rho \cdot \hat{a}_\phi = \hat{a}_\phi \cdot \hat{a}_z = \hat{a}_z \cdot \hat{a}_\rho = 0$$

$$\because \hat{a}_\rho \perp \hat{a}_\phi \perp \hat{a}_z$$

$$\hat{a}_\rho \times \hat{a}_\rho = \hat{a}_\phi \times \hat{a}_\phi = \hat{a}_z \times \hat{a}_z = 0$$

$$\left\{ \hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z, \quad \hat{a}_\phi \times \hat{a}_z = \hat{a}_\rho, \quad \hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi \right\}$$

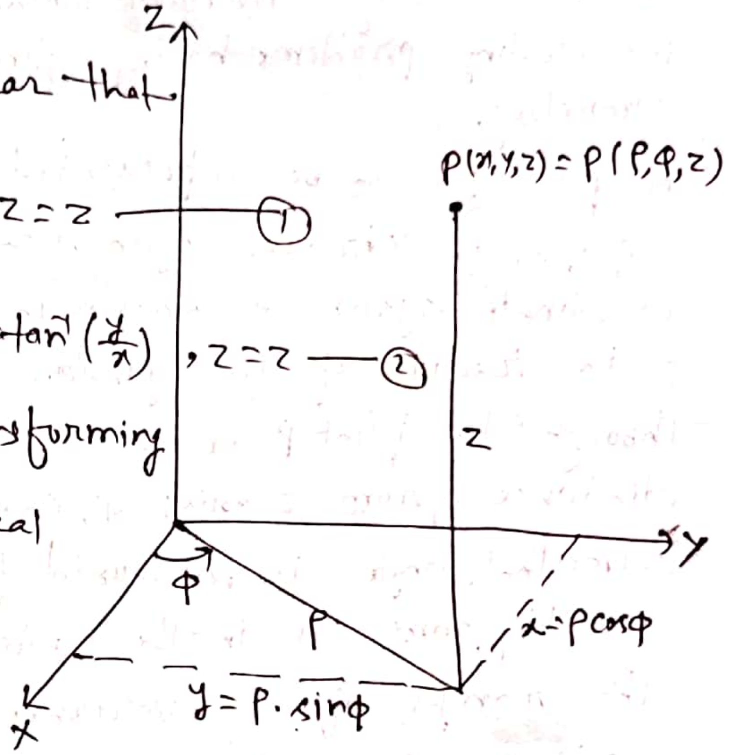
where equations under bracket are obtained in cyclic permutation.

Relation between the variables (x, y, z) of the cartesian coordinate system and the variables (ρ, ϕ, z) of the cylindrical coordinate system:

From figure, it is clear that

$$x = \rho \cdot \cos\phi, \quad y = \rho \sin\phi, \quad z = z \quad \text{——} (1)$$

$$\rho = \sqrt{x^2+y^2}, \quad \tan\phi = \frac{y}{x} \Rightarrow \phi = \tan^{-1}\left(\frac{y}{x}\right), \quad z = z \quad \text{——} (2)$$



Equ^n (1) is used for transforming a point P from cylindrical coordinate (ρ, ϕ, z) into Cartesian coordinate (x, y, z) and equ^n (2) is used for transforming a point P from cartesian coordinate (x, y, z) into cylindrical coordinate (ρ, ϕ, z).